Process Capable Tolerancing

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ABSTRACT

Just look at any engineering product design drawing revision column and it will become clear that most designs are not right the first time. Inevitably the main problems result from poor tolerance capability. The paper outlines the statistical concepts behind the process capability index $C_{pk}$ and explains its importance to the engineering designer. The paper goes on to introduce the notion of Process Capable Tolerancing (PCT) and describes a methodology for creating components and products that are robust to process variation. Software that facilitates the application of the methodology is described and its application is illustrated through an industrial case study.

(Keywords: process capable tolerancing, process capability, robust design, tolerance allocation, PCT)

Introduction

Quality costs often consume some 25% of total revenues in manufacturing business. Even the quality leaders face intimidating quality losses. The vast majority of quality costs are failure costs that include rework, scrap, warranty, product liability claims and recall costs. In general, the cost of failure is the difference between the actual production costs and what it would cost if there were no failures. This represents lost profit. Engineering change to address such issues is invariably difficult, protracted and costly. Ref. (1) and (2)

Just look at any engineering drawing revision column and it will become clear that most designs are not right the first time. Each revision is a rework of the design and many times the words REDRAWN AND REVISED appear at the top of the column. This means the previous drawing revision column had been filled and in order to provide room for subsequent revisions a new drawing was issued. There is tremendous pressure to get the drawings released, and worry about fixing them later. Most companies wait until they find a problem and then fix it. This is the “if it ain’t broke—don’t fix it” attitude. Later in
the development program when fixes are expensive, we troubleshoot the problems and are then proud of our troubleshooting skills. Clearly, a better approach is to avoid the problems in the first place. Problems must be avoided by recognizing issues early in the design phase. Many tools are available to the designer - DFA, DFM, DOE, etc. but these tools are often too cumbersome or time consuming to use. Several attempts have been made through the Six Sigma effort to compare tolerances to process variation based on historical data. There are many tolerance analysis packages that facilitate this activity. The problem is that these packages often are not user friendly and require an internal expert or outside consultant. What is needed is a fast, simple way for designers and engineers to determine whether or not the tolerances they apply are reasonable and producible.

The answer is found in Process Capable Tolerancing (PCT) – a methodology for designing components and products that are robust to process variation. The methodology involves setting process capability targets, predicting process capability levels and minimizing quality failure costs. A key element in all this is the process capability index - Cpk.

In most organizations designers and design engineers have not been involved with process control activities. As a result, the Cp and Cpk indices are not well understood in the ‘design engineering’ arena. Certainly, designers do not need to have as thorough an understanding of these indices as quality and process engineers. At the risk of offending the statisticians who might read this article, the following is a simplified explanation of Cp and Cpk aimed at illustrating the way an understanding of these terms is crucial to process capable tolerancing.

**Statistical Concepts and Cp/Cpk**

Cp is a measure of the precision of a given process. It compares the allowable deviation stated on the drawing to the process variation.

\[
Cp = \frac{\text{Design spec width}}{6\sigma}
\]

Imagine sighting in a target rifle. The marksman will usually aim at the intersection of two lines and fire off several shots. The majority of shots will often be clustered in a group. By comparing the dispersion of the majority of shots to the size of the target, the capability of the target shooting process may be determined. In the example shown in Fig. 1 on the left, if the goal is to always shoot off the head of a match, this process is not at all capable. However, if the goal is to always hit within the target on the right, it appears that the process is very capable.
An index that can be used to describe the capability of a process is \( Cp \): C for capability and p for process.

\[
Cp = \frac{\text{Design Spec Width}}{6\sigma}
\]

The symbol \( \sigma \), sigma, represents the standard deviation of a population or “A statistic used as a measure of the dispersion or variation in a distribution, equal to the square root of the arithmetic mean of the squares of the deviations from the arithmetic mean.” The standard deviation is easy to understand by looking at how it is determined in an actual case study.

A **Process Capability Case Study:**

A machine shop wished to determine the capability of their CNC turret lathe when producing external diameters of various sizes within a range. This lathe did not produce large quantities of the same part. It did however, day in and day out, produce hard, round parts for the organization. The Shop Manager decided to measure every external diameter turned on the lathe in the range from 20mm to 30mm and chart the deviation of each part from its goal even though the goal changed from part to part. Over time a large number of deviations were recorded. The results looked something like those shown in Fig. 3.
As with many manufacturing processes, this process approximated a normal distribution or bell shaped curve (Fig. 4). By using an inexpensive calculator with statistical functions, the average or mean ($\mu$) and the standard deviation ($\sigma$) may be calculated.

Notice that one $\sigma$ on the curve is where it transitions from concave to convex. It was determined that this process, for the material being used, had a $\sigma$ of 0.005mm or 0.0002in. When a process approximates the normal distribution, about 68.7% of the parts are found between $+$ and $-$ one $\sigma$ from the average. About 95% of the parts are found between $+$ and $-$ 2$\sigma$ from the average. About 99.73% of the parts are found between $+$ and $-$ 3$\sigma$ from the average.
Notice that in this example, as is true in most processes, the average is not exactly on the goal. This is where accuracy and the Cpk index will come into play.

Imagine that the mean of the process was always centered on the goal. The design engineer comes up with a terrific new design that requires a tolerance on a critical diameter be held within ±0.005mm or a total of 0.01mm.

Cp for this design would be:

\[ \text{Cp} = \frac{0.01}{6(0.005)} = 0.33 \]

This would mean that, if the process was centered on the goal, roughly 68% of the parts produced would be in tolerance, about 16% would be oversize (could possibly be reworked) and about 16% would be undersize (scrap). The wise approach would be to reject the design and send the design engineer back to the CAD system until a more robust design (one not requiring such a tight tolerance) could be created. Had the design engineer concocted a design requiring a tolerance of ±0.01, which is ±2σ, approximately 95% of the parts would be in tolerance. Or, there would be a 5% chance of parts being out of tolerance.

\[ \text{Cp} = \frac{0.02}{6(0.005)} = 0.67 \]

A tolerance of ±0.015, ±3σ, would yield approximately 99.73% in spec parts or roughly 3 discrepant parts per 1000 produced.

\[ \text{Cp} = \frac{0.03}{6(0.005)} = 1 \]

Keep in mind that there is an assumption that the process will always be centered.

When is the design robust enough? Historically Cp = 1 was considered good enough. This is a ±3σ design. Today, however, most companies claim to be trying for ±6σ. In our example the design engineer would have to create a design that would function if the critical diameter had a tolerance of ±0.03mm or ±6 times the standard deviation of 0.005mm.

A major reason why care must be taken is due to the fact that processes do not remain centered. Tools wear, machines heat up and cool down, raw material is not consistent, etc. That is where Cpk comes in.

Cpk is the index used to describe the float or drift of the distribution of parts relative to the design specifications. It is a measure of the accuracy of the process. Cpk describes how close the process mean is to the nearest limit and compares that distance to the dispersion of parts about the mean.

Using our target rifle example, accuracy is determined not by how close the average of the shots is to the center of the target but rather by how close the average is to the nearest edge
of the target (Fig. 5). If the marksman stays well clear of the edge of the target, the shots will land close to the center.

![Target Diagram](image)

Fig. 5 Process average and target limits

Cpk is valuable to the design engineer because it may be used to predict the probability of defects. In statistical terms Cpk is:

\[
Cpk = \frac{\text{Difference between the nearest spec limit and the process mean}}{3\sigma}
\]

Ref. (2)

![Cpk Diagram](image)

Fig. 6 Definition of Cpk

Design impacts the numerator of this equation since they control the limits of the tolerance. Manufacturing impacts the denominator since it is their responsibility to reduce variation and drift as much as possible.
Process Capability and Tolerancing

Designers often claim that the tolerances are determined by the function of the design. In reality, the required tolerances are determined by the design alternative selected. Functional performance is what customers always want. There are always design alternatives with some being more robust than others. In other words, some designs require tight tolerances while other design alternatives will function while allowing looser tolerances. All other things being equal the design professional should select the design that allows the largest tolerances while assuring that the functional requirements of the design are met.

In the turret lathe example, if the tolerance had been specified as ±0.03mm, which is ±6σ and the mean of the process was not allowed to drift off center by more than 1.5σ or 0.0075mm, the Cpk of the process as shown in Fig. 7 would be:

\[
\text{Cpk} = \frac{0.0225}{3 \times 0.005} = 1.5
\]

Fig. 7 CNC Turret lathe process capability

This would be considered a 6σ design. By converting Cpk to a ‘z’ score, probability tables may be used to determine that the likelihood of producing defects would be 3.4 parts per million (ppm) or less. Designers and design engineers must have a way to quickly evaluate the implications of the tolerances they assign.

Predicting Cpk when tolerances are allocated allows the designer to greatly reduce:

- Product cycle time
- Early life failures
- Scrap
Rework
Problem analysis time
Testing
Cost

The tolerancing of parts is typically a two-step process. First, tolerances are allocated to the features on the parts. Second, when there is time, these tolerances are analyzed to assure that design requirements are being met. Typically, the allocation of tolerances is based upon:

- Past practice or carryover
- Data found in handbooks
- Seat-of-the-pants guesses
- Individual experience of the designer

Sadly, none of these methods relates to the quality initiatives being driven in world-class quality companies today. All too often tight tolerances have caused problems in manufacturing that design is unaware of. Tolerances found in handbooks are frequently based on $3\sigma$ tolerancing and have not been revised to reflect today’s quality demands.

The predicted cost of a product is determined, to a large degree, by the processes and materials used to produce the parts. These materials and processes should determine the tolerances allocated. Designers need a way to instantly know if the tolerances they assign are reasonable for the processes and materials to be used.

A Methodology for Process Capable Tolerancing

The Process Capable Tolerancing methodology (PCT) requires that the designer or design engineer verify that each assigned tolerance be process capable. PCT is the one activity which links design, manufacturing and quality. This activity must be a part of any company’s “phases and gates” product development process if they are to succeed. For years quality and manufacturing areas have been gathering massive amounts of process data. This data is used to generate control charts but is seldom shared with design. Consequently, companies acquire vast amounts of data without ever acquiring any real knowledge.

The problem is compounded when suppliers are involved. Usually suppliers will tell you how good they are. Rarely will they tell you how good they are how often. Early in a program, suppliers will submit sample parts, but these parts are rarely representative of the parts you will receive in production. Design needs a tool that will tell them what to expect once the product is in full production, therefore tolerances must be determined based on real data. It is possible for a company to create its own database provided there is an understanding of how to model the data in a fashion usable to the designer. However, this is a very difficult, time consuming and costly task. Alternatively, there is now software available with a database of knowledge covering most processes used by most manufacturing companies. Ref. (3)
The PCT methodology is illustrated in Fig. 8. It is shown against the background of the product introduction process, illustrating phases of operation. The process involves setting process capability targets for the design characteristics in question, assessing process capability or allocating process capable tolerances based on process variation and assessing the effects of design geometry and material on tolerance capability.

The main objective of the methodology is the assignment of process capable tolerances to component design characteristics. Other objectives include assessing the acceptability of a design characteristic against its likely failure severity and specifying appropriate process capability (Cpk) targets for internal manufacture and external supply.

It has been assumed that for each process there is a fundamental level of inherent variability associated with processing the ‘ideal’ design under conditions of good practice in manufacturing operations. Therefore, central to the determination of $C_{pk}$ are the process capability maps that plot achievable tolerance (for an ideal design) against characteristic dimension. There are currently around 70 maps incorporated within the

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1 This common-cause or inherent variability is due to the set of factors that are inherent in a machine/process by virtue of its design, construction and the nature of its operation; for example, positional repeatability, machine rigidity, which cannot be removed without undue expense and/or process redesign. When only common cause variability is present, the process is performing at its best possible level.

2 The data given in the maps are representative of good practice in the industry concerned. It has been collected from a wide range of sources including International standards, specialist organizations, engineering texts and experimental studies. Particular maps would include as many as twenty different data sources. In all cases the data is consistent with processes that are well established and fully developed. The predicted values of $C_{pk}$ have proved to have very close correlation (98%) with SPC results.

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analysis covering processes from sand casting to honing. A sample map can be found in Fig. 9.
In addition, the analysis embodies the main design dependent factors that influence process variability. The factors included in the methodology relate to material and design geometry effects. (This kind of variation can be considered as special-cause). Assessing the degree to which the material under consideration is away from the ‘ideal’ specification is key to the analysis. (Machinability or formability ratings are examples of the data types employed in an assessment). The analysis of design geometry enables form and feature to be linked to variability during manufacture; for example, parting lines on castings and long unsupported sections. In order to assess the risk of an out of tolerance situation both a likely estimate of occurrence and a measure of failure severity are needed. Therefore, in addition to understanding the likely parts per million (ppm) defective from the Cpk estimate, the designer must consider the severity of potential failures. A technique useful in this connection is Failure Mode and Effects Analysis (FMEA). The conformability map (Fig. 10) relates FMEA Severity Rating to occurrence probability and Cpk. The map is divided into regions of acceptable design, special control and design review, and includes lines defining quality failure costs. In this way the map provides a risk assessment tool. For more information the reader is directed to references (4) and (5).

![Sample Process Capability Map](image1)

![Conformability Map](image2)

**Fig. 9 Sample Process Capability Map, Ref. (6)**
**Fig. 10 Conformability Map**

**Software support – a tolerance capability expert system (TCE)**

One software tool that is now available to design teams is TCE. This user friendly package provides a straightforward, rapid and consistent way of allocating process capable limits to design dimensions. The expert system runs on a PC under the Windows® operating environment. The memory and processor requirements are modest and the expert system can be used in either a stand-alone mode or integrated with a suite of engineering tools. The expert is coded in C++. This is a powerful general purpose
programming language, whose object-oriented nature enables the creation of a system that readily facilitates incremental expansion in the range of manufacturing processes available to the user.

The analysis begins with the selection of a manufacturing process to be investigated for the design characteristic in question and the determination of the ‘ideal’ or ‘best achievable’ value of Cpk for a given tolerance. As necessary the user can then consider the effects of design geometry and material type on process capability. (It is not always necessary to consider material effects since many of the maps have been derived for particular material types). The user is guided through a series of questions (in a wizard format) to enable the influence of geometry and material on variability to be enumerated.

Consider the Aluminum Bronze component case study (Fig. 11). The analysis on the 21.46mm critical characteristic is shown in Fig. 12. The main screen in Fig. 12 is where the user can select processes to be investigated, machining in this case. Tabs are provided to move from one process group to another. Clicking on a process displays the appropriate capability map. Double clicking the process brings up the process setup screen where the characteristic dimension and its tolerance can be entered and assessed. Note that the predicted Cpk is consistent with the shop floor value. Selecting the ‘Summary’ button on the ‘process setup’ screen gives access to the results obtained, and enables a user to derive an appropriate (Cpk) target. The failure severity rating from the FMEA can be entered and an appropriate process capability target can be obtained using coded data from the Conformability Map (Fig. 10). If the engineering requirement is not met, the software can be used to explore the effect of design changes. Changes in geometry, material, process and tolerance can be rapidly investigated.

![Fig. 11 Plunger Housing](image1)

![Fig. 12 Sample tolerance analysis](image2)

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3 Courtesy CapraTechnology Limited
Industrial case study – tolerance stack design

The case study reports on the analysis of the tolerance stack for a Smart Solenoid End Assembly (see Fig. 13). In carrying out the study the stack failure severity can be taken as $S = 4$ and the customer required a ‘worst case’ analysis. The primary design requirement was to control the plunger displacement to $0.8 \pm 0.2$ mm.

<table>
<thead>
<tr>
<th>Item</th>
<th>Negative</th>
<th>Positive</th>
<th>$+\text{Tolerance}$</th>
<th>Stat. Tol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.0000</td>
<td></td>
<td>0.0500</td>
<td>0.0025</td>
</tr>
<tr>
<td>2</td>
<td>3.0000</td>
<td></td>
<td>0.0200</td>
<td>0.0004</td>
</tr>
<tr>
<td>3</td>
<td>22.0000</td>
<td></td>
<td>0.0350</td>
<td>0.0012</td>
</tr>
<tr>
<td>4</td>
<td>8.0000</td>
<td></td>
<td>0.0200</td>
<td>0.0004</td>
</tr>
<tr>
<td>5</td>
<td>0.2000</td>
<td></td>
<td>0.0250</td>
<td>0.0006</td>
</tr>
<tr>
<td>6</td>
<td>28.0000</td>
<td></td>
<td>0.0500</td>
<td>0.0025</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Neg. Total | Pos. Total | WOW Total | SS Total
---|---|---|---
36.2000 | 37.0000 | 0.0777

Vector Sum | RSS Tol.
---|---
0.8000 | 0.2000 | 0.0875

Min Gap = 0.6000 | Max Gap = 1.0000
The analysis of the components in the tolerance stack using the TCE software is tabulated below in Table 1.

Summary of design analysis - Clearly the tolerance stack is not capable as only one characteristic is acceptable. The failure rate will be extremely high given the low values of Cpk (the associated failure cost was calculated at over $6 million per annum against product sales revenues of around $16 million). Problems with the stack resulted from confusion with the suppliers about the capability of the impact extruded Body and Magnetic Pole, and with that of the plastic molded Bobbin. In connection with the impact extrusions radial capabilities had been assumed on axial dimensions! (The software has separate process capability maps for each forming direction).

A redesign was undertaken which required focusing on improving capability and/or eliminating problematical parts from the stack.

Table 1 TCE results produced for the original design

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Process/Material</th>
<th>Dim. ±Tol.</th>
<th>Cpk(T)</th>
<th>Cpk(P)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bobbin</td>
<td>Injection mold (PBT with insert)</td>
<td>22</td>
<td>0.035</td>
<td>1.38</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>Body</td>
<td>Impact Extrusion (Forming Steel)</td>
<td>3</td>
<td>0.02</td>
<td>1.38</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>Plunger</td>
<td>Rubber molded onto M/C plunger</td>
<td>6</td>
<td>0.05</td>
<td>1.38</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>Magnetic Pole</td>
<td>Impact Extrusion (Forming Steel)</td>
<td>8</td>
<td>0.02</td>
<td>1.38</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>Tube</td>
<td>Deep Drawn (Brass)</td>
<td>0.2</td>
<td>0.025</td>
<td>1.38</td>
<td>3.62</td>
</tr>
</tbody>
</table>

Summary of Smart Solenoid Redesign - Based on the analysis results redesign solutions were generated by the design team. (See Fig. 14 for one of the solutions). The redesign shown has eliminated the Bobbin component from the stack. The Magnetic Pole (molded
into the Bobbin) has been positively located against a machined step in the Body component.
(The body was already machined at one end to provide location on the Fuel Port Block).

The TCE analysis results are tabulated in Table 2. All components are now process capable. As can be seen from Table 2 the tolerance stack is now capable and the design can be produced with low levels of failure cost. This redesign alternative resulted in a calculated failure cost of only $6 thousand per annum. (The additional machining of the Body was not significant in the calculations.)

Fig. 14 Smart Solenoid redesign
The importance of the process capability index Cpk in the design of process capable products has been illustrated and a methodology has been introduced for enabling process capable tolerancing. The approach provides a means of predicting process capability in the early stages of the design process, eliminating problems before they occur. Software developed to support the application of the technology and its role in the process of creating capable products has been overviewed. Further developments with the software will extend the coverage of geometric tolerances and include the effects of assembly/fastening processes on process capability.

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Process/Material</th>
<th>Dim. ±Tol.</th>
<th>Cpk(T)</th>
<th>Cpk(P)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bobbin</td>
<td>Injection mold (PBT with insert)</td>
<td>No longer in the tolerance stack!</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Body</td>
<td>Impact Extrusion (Forming Steel)</td>
<td>23</td>
<td>0.035</td>
<td>1.3</td>
<td>3.47 Capable</td>
</tr>
<tr>
<td></td>
<td></td>
<td>then Machined</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Plunger</td>
<td>Rubber molded onto M/C plunger</td>
<td>6</td>
<td>0.08</td>
<td>1.38</td>
<td>1.44 Capable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Magnetic Pole</td>
<td>Machined (Free-Cutting Steel)</td>
<td>6</td>
<td>0.02</td>
<td>1.38</td>
<td>3.4 Capable</td>
</tr>
<tr>
<td>7</td>
<td>Tube</td>
<td>Deep Drawn (Brass)</td>
<td>0.2</td>
<td>0.015</td>
<td>1.38</td>
<td>3.13 Capable</td>
</tr>
</tbody>
</table>

Table 2 TCE results for Smart Solenoid redesign

**Concluding Remarks**

The importance of the process capability index Cpk in the design of process capable products has been illustrated and a methodology has been introduced for enabling process capable tolerancing. The approach provides a means of predicting process capability in the early stages of the design process, eliminating problems before they occur. Software developed to support the application of the technology and its role in the process of creating capable products has been overviewed. Further developments with the software will extend the coverage of geometric tolerances and include the effects of assembly/fastening processes on process capability.

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